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## Integrated Robust Airline Schedule Development

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### Abstract

In air transportation, airline profitability is influenced by the airline's ability to build flight schedules. In order to generate operational schedules, airlines engage in a complex decision-making process, referred to as airline schedule planning. Up to now, the generation of flight schedules has been separated and optimized sequentially. The schedule design has been traditionally decomposed into two sequential steps. The frequency planning and the timetable development. The purpose of the second problem of schedule development, fleet assignment, is to assign available aircraft types to flight legs such that seating capacity on an assigned aircraft matches closely with flight demand and such that costs are minimized. Our work integrates these planning phases into one single model in order to produce more economical solutions and create fewer incompatibilities between the decisions. We propose an integrated robust approach for the schedule development step. We design the timetable ensuring that enough time is available to perform passengers' flight connections, making the system robust avoiding misconnected passengers. An application of the model for a simplified IBERIA network is shown.

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**Keywords:** Robust airline schedule; timetable planning; fleet assignment.

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### 1. Introduction

The airline schedule planning problem is defined as the sequence of decisions that need to be made to make a flight schedule operational. Given the high level of competition in the airline industry, effective decision making is crucial to the profitability of an airline. This is the motivation for this paper in which we focus on the integration of the decision making process. Our goal is to achieve simultaneous rather than sequential solution; a simultaneous solution will generate more economical solutions and create fewer incompatibilities between the decisions. Moreover, with the integration of the different planning process phases a greater robustness degree may be achieved, obtaining smoother solutions, which in case of incident may be recovered in an easier way.

There are three major components in the schedule development step. The first step, the schedule design, is arguably the most complicated step of all. Traditionally the schedule design has been decomposed into two sequential steps. First, the frequency planning, in which planners determine the appropriate service frequency in a market; and the second one, the timetable development, in which planners place the proposed services throughout the day, subject to network consideration and other constraints. The purpose of the second step of schedule

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development, fleet assignment, is to assign available aircraft types to flight legs such that seating capacity on an assigned aircraft matches closely with flight demand and such that costs are minimized. Then, the network is decomposed into different networks, each one corresponding to a particular fleet type. Given these networks, the assignment of individual aircraft to flight legs is done in the aircraft maintenance routing step, the third step. Crew scheduling involves the process of identifying sequences of flight legs and assigning both the cockpit and cabin crews to these sequences.

Designing an airline network is an extremely complex task due to the huge number of variables affecting the design, i.e. passenger demand, ground facilities and capacity, competition, etc. These issues are not always easily modeled and usually result in huge models.

In hub-and-spoke networks, connecting passengers are very common. In order to fly from one spoke to a different one, a connection must be performed in the network hub. The time needed to accomplish these connections is not well known and it depends on different aspects as congestion, delays, etc. Thus, robustness will be introduced through passengers' itineraries, providing them enough connection time. However, as connection time increases, passengers' dissatisfaction and low resource utilization may also increase. Therefore, robustness is introduced avoiding misconnected passengers but accounting for passengers' costs and fleet utilization.

This robustness criterion may have different impact over different aspects. First of all, increasing connection time means that passengers will have to perform longer connections, increasing their dissatisfaction due to the longer connection time. However, the probability of being misconnected will be ameliorated. Another issue might be fleet utilization. Designing the timetable providing robust itineraries without accounting for the fleet will surely provide a lower utilization of it. Nevertheless, this hassle may be controlled by facing the problem in an integrated way, accounting for fleet resources as we will propose later in the paper. An important phase in airline planning is the crew scheduling. In this paper we do not address this problem. However, the new approach we propose will affect this planning phase. As we will provide longer connection time for passengers, it may also occur the same for crews. This could result in the necessity of having more crews to operate the schedule.

The problem presented in this paper consists of determining the schedule design and the fleet assignment simultaneously. For this purpose, average demand values will be used and for each market demand, its disaggregation in time will be considered. In supply's side, a space-temporal graph will be used. As different fleet types will be considered the space-temporal graph must be replicated for each fleet type.

## **2. State of the Art**

Soumis et al. (1980) consider the problem of selecting passengers that will fly on their desired itinerary with the objective of minimizing spill costs. Flight schedules are optimized by adding and dropping flights. They consider the passenger flow process by assigning dissatisfaction costs for unattractive itineraries and passenger spill costs for loading good itineraries.

Armacost et al. (2002) describe a new approach for solving the express shipment service network design problem. They transform conventional formulations to a new formulation using what they term composite variables. The formulation relies on two key ideas: first, they capture aircraft routes with a single variable, and second, package flows are implicitly built into the new variables, the composite variables.

Lan et al. (2006) consider passengers who miss their flight legs due to insufficient connection time. They develop a new approach to minimize passenger misconnections by retiming the departure times of flight legs within a small time window. They present computational results using data from a major U.S. airline and showing that misconnected passengers can be reduced without significantly increasing operational costs.

Jiang and Barnhart (2009) propose a dynamic scheduling approach that reoptimizes elements of the flight schedule during the passenger booking process. They recognize that demand forecast quality for a particular departure date improves as it approaches; thus, they redesign the flight schedule at regular intervals, using information from both revealed booking data and improved forecasts.

Lately, researchers have focused on determining incremental changes to flight schedules, producing a new schedule by applying a limited number of changes to the existing schedule. Lohatepanont and Barnhart (2004), in their incremental optimization approach select flight legs to include in the flight schedule and simultaneously optimize aircraft assignments to these flight legs. Garcia (2004) extends the previous model and proposes a combination between it and a decision time window model. Kim and Barnhart (2007) consider the problem of

designing the flight schedule for a charter airline. Exploiting the network structure of the problem, they develop exact and approximate models and solutions, and compare their results using data provided by an airline. Espinoza et al. (2008) present an integer multicommodity network flow model with side constraints for on-demand air transportation services.

Cadarso and Marín (2010) propose a multiobjective integrated robust approach for the schedule design phase, deciding jointly flight frequencies and timetable. The objectives are passengers' satisfaction and operator costs. They design the timetable ensuring that enough time is available to perform flight connections, making the system robust avoiding misconnected passengers.

In this paper, a new integrated approach to solve the schedule development is presented. Up to now, it has been solved sequentially. This sequential approach provides local optimum for each subproblem but not for the overall planning. Moreover, it could be that there are not enough resources to operate a given timetable. In such a case, planners must face an iterative process until they obtain a feasible planning but probably not being the overall optimum. The approach proposed in this paper will provide a global optimum to the schedule development phase overcoming the mentioned deficiencies. In addition, robustness will be introduced for passengers' connections. This will probably mean longer connection times but as we are also accounting for fleet resources it does not necessarily mean lower fleet utilization.

### 3. Problem Description

In this paper the schedule design and fleet assignment problems are treated in an integrated way. Frequencies and departure times must be determined for every itinerary attending each market. Moreover, fleet types must be assigned to every flight leg.

Given the estimated demand for travel, an airline wishes to determine the flight schedule which maximizes its profit while taking into account the satisfaction of its customers. In this system, two agents interact: the aircraft flow in the physical network, and the passengers using flight legs to travel.

The network is built considering the airports associated to the demand to be met. It is formed by the airports and all the feasible airway or sections alternatives linking them. The airports are defined by the operations that can be performed within them. They are characterized by available capacity (i.e., slots) for landing and taking off determined by airports' operators.

The sections are the links between the airports. Each section is characterized by an origin airport and a destination airport. When a section is flown it will be called flight leg; a flight leg is defined by an origin, destination and a departure time, that is, a flight leg is defined by the pair  $(s, t)$ , where  $s$  is an element of the sections' set  $S$ , and  $t$  in  $\{0, 1, \dots, T-1\}$  is the departure time from the origin of  $S$ . The set of all possible flight legs is  $F = S \times \{0, 1, \dots, T-1\}$ .

In this way, we will consider that the time is discretized by partitioning the planning period,  $T$ , into intervals of equal length with starting points  $0, 1, \dots, T-1$ . The intervals' length will be taken as the time unit.

For this work the unconstrained demand is characterized by the origin,  $o$ , and destination,  $d$ , airports. Each pair  $(o, d)$  is mentioned as the market  $w$ . For each pair  $w$  the demand of passengers  $d_w$  is assumed fixed and known datum. This demand is disaggregated in time. However, departure time is not a fixed value, passengers will accept without additional cost a departure time from a set of desired departure times in each market  $(T_w)$ . In this way, it is supposed that passengers have a feasible set of departure times.

For each demand, passengers from origin to destination are considered in all possible itineraries  $i$ , that may be classified by pair  $w$  as  $I_w$ . Each itinerary is defined by a set of sections that connect different airports. It can be composed of one section or more than one including in this last case intermediate stops at different airports. In this way, passenger dissatisfaction with connecting time will be considered. As connecting time grows, passenger dissatisfaction rises up. However, if connecting time is not enough, passengers may be misconnected. In order to avoid misconnected passengers as much as possible, robust itineraries will be introduced.

As competition effects are not considered in this problem, every flight leg will probably be crowded because the unconstrained market demand is considered. However, this situation is far from the real one. Due to competing airlines, unconstrained demand will be divided between different flight legs. In order to represent this issue, the capacity offered for each flight leg will not be the entire one but the one obtained by an average load factor. This

average load factor will be obtained from the airline data. Besides, competition within the same airline will be avoided by imposing a separation time between flight legs operating the same origin and destination.

#### 4. Robustness

As mentioned before, robustness is introduced through passengers in itineraries with more than one flight, where a connection is mandated. Adding more slack for connection can be good for connecting passengers, but can result in reduced productivity of the fleet; the challenge then is to determine where to add this slack so as to maximize the benefit to passengers without getting worse the network operation (Lan et al. (2006)).

Every connection is characterized by the minimum time required to perform it. This time varies from airport to airport and it can also vary along the day. In this way, in itineraries with more than one flight, every passenger is mandated a minimum connection time (*mct*) for flight connections. However, this time will not be always enough to perform the connection.

We assume that the number of disrupted passengers depends on the available time to perform the flight connection. Once flights' arrival (*at*) and departure (*dt*) times are known, the available connection time (*ct*) is also known. From airlines' historical data, disrupted passengers number variability with connection time may be known.

Assigning a statistical distribution to misconnected passengers, the probability of getting misconnected passengers depending on connection time can be calculated. The exponential distribution has been chosen, that is, the number of misconnected passengers will decrease exponentially with the available excess connection time. If the available connection time is negative, every connecting passenger will be misconnected. The probability distribution of having misconnected passengers is as follows:

$$f(ect) = \lambda_{i,t,t'} e^{-\lambda_{i,t,t'}(ect+t_0)} \quad \forall ect \geq 0$$

, where  $\lambda_{i,t,t'}$  depends on the itinerary connection characteristics and is chosen adjusting the probability distribution to historical data; it is supposed that once the connection characteristics are known, the assigned gates will be probably known due to historical availability. *ect* represents the available excess connection time ( $ect = ct - mct$ ).  $t_0$  is a location parameter to fit the distribution. In this way, given the available excess connection time (*ect*), the probability of having misconnected passengers ( $prob_{i,t,t'}$ ) is:

$$prob_{i,t,t'} = e^{-\lambda_{i,t,t'}(ect+t_0)} \quad \forall ect \geq 0$$

Once misconnected passengers are known, they must be removed from the remaining flights of the itinerary, so extra capacity arises in those flights making possible to accommodate other passengers in it in case of disrupted passengers.

#### 5. Integrated Robust Airline Scheduling Model

In the proposed model the entire demand satisfaction is not enforced. The neglected demand is penalized in the objective function. Passengers transfer possibility is considered, that is, for every passenger itinerary the possibility of intermediate stops in the flight are taken into account. Itineraries composed of up to two flight legs are considered.

In order to avoid full flights, an average load factor is introduced. These full flights may occur because competing airlines are not considered in the model formulation. To overcome with this deficiency, the average load factor will determine the maximum attainable passenger demand in each flight leg.

We suppose that the schedule will be periodic, that is, the schedule will repeat after the planning period ends. For this purpose, we must take care about airplanes location at the end of the planning period. Its location must be the necessary one to repeat the schedule.

The following notation is introduced to explain the Integrated Robust Airline Scheduling Model (IRASM):

- Sets:

$P(p)$ : fleet types' set.

$T(t)$ : periods' set.

$S(s)$ : sections' set. Each section is defined by an origin and a destination.

$W(w)$ : markets' set. Markets are defined by the origin, destination, and the departure time ( $o, d, at_w$ ).

$K(k)$ : airports' set.

$I(i)$ : itineraries' set.

$I2(i) \subset I(i)$ : itineraries' set composed of more than one section.

$I_w$ : itineraries' set attending market  $w$ .

$W_i$ : markets' set attended by itinerary  $i$ .

$T_w$ : periods' set for each market  $w$ .

$I1_s$ : itineraries' set containing section  $s$  as first section.

$I2_s$ : itineraries' set containing section  $s$  as second section.

$AS_k$ : sections' set arriving at airport  $k$ .

$DS_k$ : sections' set departing from airport  $k$ .

$TS_{i,t} \subset T(t) = \{t' / t' \geq t + \sum_{s \in I1_s} st_{s,t} + mct_{i,t}\}$ : feasible departure time set for the second flight leg in

itineraries with more than one flight leg.

$CT_i$ : count time.

- Parameters:

$c_{s,t}^p$ : operating cost in section instance  $(s,t)$  with  $p$  fleet type.

$q_p$ : passenger capacity in each fleet type  $p$ .

$b_{s,t',t}$ : 1, if flight leg  $(s,t')$  is flying during time period  $t$ .

$N_p$ : fleet size for each fleet type  $p$ .

$t_i, t_f$ : initial and final periods in the planning period.

$\tau_{i,t,t'}$ : passengers' dissatisfaction due to transshipments times in itinerary  $(i,t)$  with more than one section, departing the second one during  $t'$ .

$dpc_w$ : cost per disrupted passenger from market  $w$ .

$dpc_i$ : cost per disrupted passenger in itinerary  $i$  due to lack of time to perform transshipments.

$qa_{k,t}$ : maximum airplane arrival capacity of airport  $k$  during each time period  $t$ .

$qd_{k,t}$ : maximum airplane departure capacity of airport  $k$  during each time period  $t$ .

$q_{s,t}$ : maximum airplane capacity in each section  $s$  and period time  $t$ .

$st_s$ : minimum separation time between two consecutive departures of section instances  $s$  (in periods).

$d_w$ : passenger unconstrained demand for each market  $w$ .

$st_{s,t}$ : section instance  $(s,t)$  trip time. We include the section trip time duration dependent on departure time; this is due, i.e., to congested airports or weather conditions which may obey to slow down the airplane.

$mct_{i,t}$ : minimum connection time for each itinerary  $i$  departing during time period  $t$ .

$prob_{i,t,t'}$ : likelihood of passengers from itinerary  $i$  departing during  $t$  being misconnected in their flight connection with the second flight departing during  $t'$ .

$alf_p$ : average load factor for each fleet type  $p$ .

- Variables:

$z_{s,t}^p$ : =1, if section  $s$  departs during  $t$  period with  $p$  fleet type; 0, otherwise.

$yt_{k,t,p}$ : integer variable. It represents grounded material of type  $p$  at airport  $k$  and during time period  $t$ .

$h_{i,t}^w$ : integer variable. Passengers in itinerary  $i$  and market  $w$  departing during  $t$  period.

$sh_{i,t,t'}$ : integer variable. Passengers using itinerary  $i \in I2(i)$  departing during period time  $t$ , and the second section during period time  $t'$ . This auxiliary variable represents passengers using a flight leg after a transfer.

$dp_w$ : integer variable. Disrupted passengers from market  $w$ .

### 5.1. Objective Function

$$\text{Min } z = \sum_{s \in S} \sum_{t \in T} \sum_{p \in P} c_{s,t}^p z_{s,t}^p + \sum_{w \in W} dpc_w dp_w + \sum_{i \in I2} \sum_{t,t' \in T} dpc_i prob_{i,t,t'} sh_{i,t,t'} + \sum_{i \in I2} \sum_{t,t' \in T} \tau_{i,t,t'} sh_{i,t,t'} \quad (1)$$

The objective ( $z$ ) function (1) accounts for operator and passengers costs. The first term represents operating costs, the second one incurred costs due to disrupted passengers, that is, spill costs, and the following one, costs due to lack of time to perform transshipments. These costs are considered as operator's costs. The last term in the objective function represents passengers' dissatisfaction costs with flight connections.

Operating costs are the costs the company incurs due to the operation of flight legs. We calculate these costs as the product of the number of block hours in each section  $s$  departing during period  $t$  with a determined fleet type  $p$  ( $NBH_{s,t}^p$ ) by the cost per block hour for each fleet type ( $cBH_p$ ). We compute these costs in (2).

Disrupted passengers are passengers that the company does not attend due to lack of capacity or high dissatisfaction costs. We can consider them as spill costs, that is, lost revenue. These costs can be computed as the distance the passengers would go if they were attended by the spill cost.

$$c_{s,t}^p = cBH_p NBH_{s,t}^p \quad (2)$$

$$E[mp]_{i,t,t'} = prob_{i,t,t'} sh_{i,t,t'} \quad (3)$$

$$\tau_{i,t,t'} = K \left( t' - t - \sum_{s \in I1_s} st_{s,t} \right) \quad (4)$$

We minimize the number of disrupted passengers due to misconnections. In this way, we introduce the robustness criterion defined above. The number of expected misconnected passengers ( $E[mp]_{i,t,t'}$ ) is calculated in (3).

Passengers' costs (4) are composed of the dissatisfaction. This term measures whether the itinerary is composed of more than one flight leg or not. The constant K may be calibrated through passengers surveys and transform the time units into costs units.

The objective function is subject to the following groups of constraints:

### 5.2. Passengers Constraints

$$\sum_{i \in I_w} \sum_{t \in T_w} h_{i,t}^w = d_w - dp_w \quad \forall w \in W \quad (5)$$

$$\sum_{w \in W_i} h_{i,t}^w = \sum_{t' \in TS_{i,t}} sh_{i,t,t'} \quad \forall i \in I2, t \in T \quad (6)$$

$$\sum_{i \in I1_s} \sum_{w \in W_i} h_{i,t}^w + \sum_{i \in I2_s} \sum_{t' \in T} (1 - prob_{i,t,t'}) sh_{i,t,t'} \leq \sum_{p \in P} alf_p q_p z_{s,t}^p \quad \forall s \in S, t \in T \quad (7)$$

Constraints (5) ensure the passenger demand allocation through available itineraries in the network; they account for disrupted passengers. Group of constraints (6) ensures that passengers in two sections itineraries remain in their trip during the second section; they also consider the average necessary time for performing transshipment. Constraints (7) ensure that there are enough active sections or flight legs to satisfy the passengers flow; misconnected passengers are removed from the flight leg.

### 5.3. Flight Leg and Airport Constraints

$$\sum_{p \in P} z_{s,t}^p \leq 1 \quad \forall s \in S, t \in T \quad (8)$$

$$\sum_{t' \in T} \sum_{p \in P} b_{s,t',t} z_{s,t'}^p \leq q_{s,t} \quad \forall s \in S, t \in T \quad (9)$$

$$\sum_{t' \in T} \sum_{p \in P} z_{s,t'}^p \leq 1 \quad \forall s \in S, t \in T \quad (10)$$

$$\sum_{s \in AS_k} \sum_{t' \in T} \sum_{p \in P} z_{s,t'}^p \leq qa_{k,t} \quad \forall k \in K, t \in T \quad (11)$$

$$\sum_{s \in DS_k} \sum_{p \in P} z_{s,t}^p \leq qd_{k,t} \quad \forall k \in K, t \in T \quad (12)$$

Constraints (8) ensure that only one unique fleet type can be assigned to each flight leg. Constraints (9) are section capacity constraints; they ensure that the number of aircraft in a section at each period is lower than a maximum number; this capacity may depend on air navigation systems and regulations. Group of constraints (10) ensures that the same flight leg does not depart until a specified time has been spent; in this way, competition between flights from the same airline is avoided. Constraints (11)-(12) are airport capacity constraints that try to



spare the departures and arrivals at airports at each period; this is mandated by the available slots in the airport to land or take off. Depending on the time, these slots may vary in costs. These costs are included in the operating costs.

#### 5.4. Fleet Flow, Capacity and Symmetry Constraints

$$yt_{k,t-1,p} + \sum_{s \in AS_k} \sum_{\substack{t' \in T \\ t=t'+st_{s,d}}} z_{s,t'}^p = yt_{k,t,p} + \sum_{s \in DS_k} z_{s,t}^p \quad \forall k \in K, t \in T, p \in P \quad (13)$$

$$\sum_{t' \in T} b_{s,t',t} z_{s,t'}^p + \sum_{k \in K} yt_{k,t,p} \leq N_p \quad \forall t \in CT, p \in P \quad (14)$$

$$yt_{k,t_i,p} = yt_{k,t_f,p} \quad \forall k \in K, p \in P \quad (15)$$

Block of constraints (13) are the flow conservation equations for each airport and fleet type. Constraints (14) are the fleet capacity constraints; we must count the necessary aircraft to perform the schedule and compare it to the available ones. Constraints (15) state that the network must be symmetric in order to repeat the same schedule once the planning period has ended.

#### 5.5. Variable Dominion

$$z_{s,t}^p \in \{0,1\} \quad \forall s \in S, t \in T, p \in P \quad (16)$$

$$yt_{k,t,p} \in \mathbb{Z}^+ \quad \forall k \in K, t \in T, p \in P \quad (17)$$

$$dp_w \in \mathbb{Z}^+ \quad \forall w \in W \quad (18)$$

$$h_{i,t}^w \in \mathbb{Z}^+ \quad \forall i \in I, t \in T, w \in W \quad (19)$$

$$sh_{i,t,t'} \in \mathbb{Z}^+ \quad \forall i \in I2, t, t' \in T \quad (20)$$

Constraints (16)-(20) define the variable dominion. We have defined passenger variables as integer positive variables. However, as the demand number is an average value, these variables dominion may be relaxed to positive variables. In addition, variables  $yt_{k,t,p}$ , being grounded airplanes at airports, are defined as a sum of binary variables  $z_{s,t}^p$ ; in this way, its definition dominion might be relaxed to positive variables as well.

## 6. Computational Experiments

As a proof of the model we have done some computational experience. We have implemented a simplified version of IBERIA's air network (Figure 1): the Spanish network. It is a pure hub-and-spoke network with 23 different airports. There are three different fleet types available for this study case: 23 A-319 with 141 seats each, 35 A-320 with 171 seats each and 19 A-321 with 200 each. Some of these data are publicly available in IBERIA's web page (IBERIA).

The planning period we have considered is 24 hours. We require the planning to be periodic, that is, the fleet distribution must be equal at the beginning and the ending of the planning period.

The model size for this study case is shown in Table 1. In this case time has been discretized into periods of 15 minutes. We have considered every potential flight leg between each spoke and the hub.

Our runs have been performed on a Personal Computer with an Intel Core2 Quad Q9950 CPU at 2.83 GHz and 8 GB of RAM, running under Windows 7 64Bit, and our programs have been implemented in GAMS 23.2/Cplex 12.



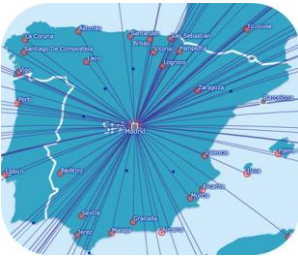


Figure 1: Hub-and-Spoke network

As it was explained above, robustness is achieved through passengers that must perform flight connections. In order to demonstrate that a more robust schedule is obtained using the proposed approach, a comparison is made with a non-robust Integrated Airline Scheduling Model (IASM). The IASM is the same model explained above but removing robustness aspects, that is, the objective function's term in (1) penalizing misconnected passengers, and the terms in constraints (7) accounting for misconnected passengers in flight leg's capacity.

Table 1. IRASM size: number discrete and continuous variables, constraints and non-zero elements

Discrete Variables	Continuous Variables	Constraints	Non-zeroes
11349	636762	100914	1820521

Table 2. Comparison between non-robust (IASM) and robust (IRASM) models

	IASM	IRASM
Misconnected Passengers (%)	1.64	0.258
Neglected Passengers (%)	64	64
Dissatisfaction Costs ( $\times 10^3$ )	1763.4	2031.4
Operating Costs ( $\times 10^3$ )	2433.3	2412.7
Objective Function ( $\times 10^3$ )	6446.23	6456.59

In Table 2 a summary of the results is presented. In the first row, misconnected passengers are compared for the non-robust and robust cases. The percentage of expected misconnected passengers is shown; this percentage is calculated with respect to the total number of attended passengers. For the robust case the percentage is sensitively reduced. In the second row, the percentage of neglected passengers is presented for both cases. Robustness is achieved through the reduction in the expected number of misconnected passengers. However, this reduction is not for free, it has a price: the robustness price. Passengers' dissatisfaction costs are written in the third row. One must note that these costs are greater in the robust case (IRASM), that is, in the robust approach where misconnected passengers number has been reduced, passengers' dissatisfaction has been increased. This is due to the fact that ameliorating the number of misconnected passengers means increasing connection time. Therefore, supposing that passengers' dissatisfaction costs increase linearly with connection time, we have that a trade-off must be found between these costs. Operating costs are shown in the fourth row. Operating costs are similar for both cases. In the last row, the objective function value is shown. In the robust case (IRASM), objective function's value is greater than the no robust one (IASM).

The computational times in seconds for IASM and IRASM in the proposed network were 456.85 and 542.02, respectively.

## 7. Conclusions

A new robust approach has been proposed to solve the airline scheduling problem, where schedule design and fleet assignment problems are jointly solved. In addition, passengers' flows are obtained through different itineraries in the network.

Robustness has been introduced through itineraries with more than one flight leg. When an intermediate stop

must be performed, passengers need some undetermined time to accomplish it. This undetermined time is captured through statistical distribution and it is introduced into the model to represent expected misconnected passengers. In this way, the expected costs that the operator would incur due to misconnected passengers are reduced.

The model has been tested in a IBERIA's simplified network. Computational results show how robustness may be achieved. However, this robustness has a price. The robust approach has been compared with a no robust approach showing the price of the achieved robustness.

Further research may include the introduction of competition effects in the model formulation. In this way, the obtained market share would depend on competing airlines.

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